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PRE- AND POST-COMPENSATION TO ACHIEVE SYSTEM TYPE
IN LINEAR MULTIVARIABLE SERVOMECHANISMS*

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ABSTRACT

Methods for specifying pre- and post-compensation to achieve a desired system type in linear multivariable servomechanisms are developed. Sufficient conditions are derived for both forms of compensation when the compensator is restricted to a diagonal input-output representation, and a method for realizing reduced-order pre-compensators is presented for the case where the restriction is removed.

I. INTRODUCTION

The basic theory of system type for linear multivariable (m -input/ m -output) servomechanisms has been developed in [1-4]. There, system type is expressed as an m -vector of integers which describes the steady-state tracking properties of each input-output pair of the closed-loop system as a function of certain characteristics of the open-loop system. In order to exploit this theory, it is necessary to have systematic compensation methods which can be used to change a system of a given type to one of a different (higher) type to satisfy performance specifications. In a logical sequel to [1], the present paper addresses this issue of compensation. Some work along these lines has been presented in [4], but the work here provides a more general and comprehensive treatment.

In contrast to the single-input/single-output case where rendering a system type ℓ is relatively straightforward [5], the multivariable case requires consideration of both pre-compensation and post-compensation. This is due to the fact that rational transfer function matrices do not commute whereas scalar transfer functions do. A distinction is thus made throughout the presentation between these two forms of compensation.

Briefly, the content of the paper is the following. In Section II, results are developed which permit a designer to change system type using simple diagonal transfer function matrix compensators. Sufficient conditions for attaining any specified type with both pre- and post-compensators are given. Also in this section is a simplified proof of the controllability condition of [6, 7] for a

multivariable servomechanism which employs pre-compensation. Section III deals with reduced-order pre-compensation. The main result there is an existence theorem whose proof is constructive. The conclusion is given in Section IV.

II. SIMPLE COMPENSATORS

The first method of creating an open-loop transfer function matrix of specified type is the simplest and is called post-compensation. The reason for the name is obvious from Fig. 1.

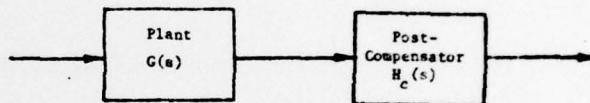


Fig. 1. Post-compensation.

If $H_c(s)$ is restricted to be diagonal, the solution of the compensation problem is characterized as follows.

Proposition 1. Let $G(s)$ be an $m \times m$ type $[\ell_1 \ \ell_2 \ \dots \ \ell_m]$ transfer function matrix. In order that the compensated system, $G_c(s) = H_c(s) G(s)$ be type $[\ell'_1 \ \ell'_2 \ \dots \ \ell'_m]$ where $\ell'_i \geq \ell_i$ for every i , $1 \leq i \leq m$, it is sufficient that

$$H_c(s) = \text{diag} \left[\frac{1}{s - \ell'_1} \ \frac{1}{s - \ell'_2} \ \dots \ \frac{1}{s - \ell'_m} \right]$$

Proof. With the given $H_c(s)$,

$$\begin{aligned} G_c(s) &= H_c(s) G(s) \\ &= H_c(s) H(s) G'(s) \end{aligned}$$

where, from the definition [1, 2] of a type $[\ell_1 \ \ell_2 \ \dots \ \ell_m]$ transfer function matrix,

$$H(s) = \text{diag} \left[\frac{1}{s - \ell_1} \ \frac{1}{s - \ell_2} \ \dots \ \frac{1}{s - \ell_m} \right]$$

and $G'(s)$ has no zeros at the origin. Thus $G_c(s) = H(s) G'(s)$ with

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$$\hat{H}(s) = H_c(s) H(s)$$

$$= \text{diag} \left[\frac{1}{s_1}, \frac{1}{s_2}, \dots, \frac{1}{s_m} \right]$$

so that the compensated system is of the desired type. \square

In general, this configuration requires the fewest number of integrators. However, using the results of Section III, it can be shown that there are cases where precompensation of lower order exists.

Due to the nature of the outputs of many systems, post-compensation may be undesirable or even impossible to implement. In this case one may use pre-compensation as indicated in Fig. 2 to produce a transfer function matrix of specified type. The



Fig. 2. Pre-compensation.

structure here is, of course, identical to that of classical servomechanism theory. The following provides a sufficient condition for this configuration when $H_c(s)$ is again restricted to be diagonal.

Proposition 2. Let $G(s)$ be an $m \times m$ type $[\ell_1 \ell_2 \dots \ell_m]$ transfer function matrix. In order that the compensated system $G_c(s) = G(s) H_c(s)$ be at least type $[\ell'_1 \ell'_2 \dots \ell'_m]$ where $\ell'_i > \ell_i$ for at least one i , $1 \leq i \leq m$, it is sufficient that

$$H_c(s) = \text{diag} \left[\frac{1}{s^L}, \frac{1}{s^L}, \dots, \frac{1}{s^L} \right]$$

where H_c is $m \times m$ and $L = \max_i (\ell'_i - \ell_i)$.

Proof. The desired result is an immediate consequence of Theorem 4 of [1] with $G(s) = G_1(s)$ and $H_c(s) = G_2(s)$. \square

This is the form of compensator which was used by Young and Willems [6], and Porter and Bradshaw [7]. Their approach employed the state-space description of the plant with state variable feedback. Although not explicitly so stated, their solution was based on the fact that the plant was at least type $[0 \ 0 \ \dots \ 0]$ since their final condition for controllability is equivalent to the plant having no zeros at the origin.

The theorem given below is an extension of the basic result in [6]. The present proof is new and much simpler than the one in [7].

Consider the system

$$\dot{x} = \hat{A}x + \hat{B}u(t) + Gy_d(t) \quad (1)$$

where

$$\hat{x} = [x \ z_1 \ z_2 \ \dots \ z_r]^T$$

$$\hat{A} = \begin{bmatrix} A & 0 & 0 & \dots & 0 & 0 \\ -C & 0 & 0 & \dots & 0 & 0 \\ 0 & I & 0 & \dots & 0 & 0 \\ 0 & 0 & I & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & I & 0 \end{bmatrix}$$

$$\hat{B} = [B \ 0 \ 0 \ \dots \ 0]^T$$

and

$$G = [0 \ I \ 0 \ \dots \ 0]^T$$

where superscript T denotes the transpose. In these relations, A, B and C are, respectively, $n \times n$, $n \times r$ and $m \times n$; the z_i are m -vectors; $u(t)$ is an accessible input to the system; $y_d(t)$ is an m -dimensional command input; and I and 0 are, respectively, the identity matrix and null matrices of suitable order. It is shown in [7] that if there exists an $r \times (n + mr)$ matrix K such that $\hat{A} + BK$ has open left half-plane eigenvalues, then for

$$y_d(t) = \begin{bmatrix} a_1 \\ \vdots \\ a_r \\ \vdots \\ a_m \end{bmatrix} t^{r-1}$$

where $t \geq 0$ and the a_i are arbitrary real constants, the tracking error $e(t) = y_d(t) - y(t) = y_d(t) - Gx(t)$ has the property that

$$\lim_{t \rightarrow \infty} e(t) = 0 \quad (2)$$

The resulting m -input/ m -output closed-loop system thus becomes a type $[r \ r \ \dots \ r]$ multivariable servomechanism. It is clear that the condition in Eq. (2) can be achieved if (\hat{A}, \hat{B}) is controllable.

Theorem 1. (\hat{A}, \hat{B}) is controllable if and only if

(i) (A, B) is controllable

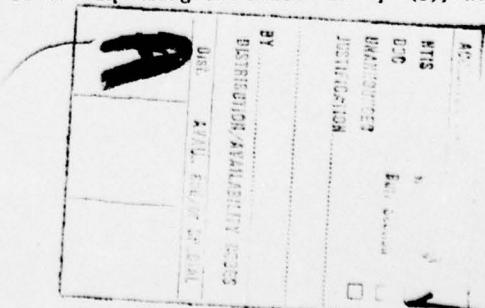
and

$$(ii) \text{rank} \begin{bmatrix} A & B \\ -C & 0 \end{bmatrix} = n + m$$

Proof. From [8], (\hat{A}, \hat{B}) is controllable, if and only if

$$\text{rank} [sI - \hat{A} \mid \hat{B}] = n + mr \quad (3)$$

for all s . Expanding the matrix in Eq. (3), one has



$$[sI - \hat{A} \quad \hat{B}] = \begin{bmatrix} sI - A & 0 & 0 & \dots & 0 & 0 & B \\ 0 & sI & 0 & \dots & 0 & 0 & 0 \\ 0 & -I & sI & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & sI & 0 & 0 \\ 0 & 0 & 0 & \dots & -I & sI & 0 \end{bmatrix}$$

For $s \neq 0$, the last m rows are clearly independent, and independent of the first n rows. Therefore, if $\text{rank } [sI - A \quad B] = n$, Eq. (3) would be satisfied for $s \neq 0$. However, this is the case by condition (i). Conversely, if condition (i) is not satisfied, $\text{rank } [sI - A \quad B]$ is strictly less than n and the system is not controllable.

For $s = 0$, the last $(m-1)r$ rows are independent, and independent of the first $n+m$ rows. By condition (ii), the first $n+m$ rows are independent so that Eq. (3) is again satisfied. Conversely, if condition (ii) is not satisfied, then Eq. (3) will not hold for $s = 0$.

Thus Eq. (3) holds if and only if conditions (i) and (ii) are met. This is equivalent to the controllability of (\hat{A}, \hat{B}) . \square

The fact that Eq. (1) describes a type $[r \ r \ \dots \ r]$ system can also be shown via the application of Theorem 3 of [1] and Proposition 2 above.

III. REDUCED-ORDER PRE-COMPENSATION

The results of the preceding section show that unity feedback systems of a specified type may be synthesized using diagonal pre- or post-compensation. The sufficient condition of Proposition 2 gives an upper bound on the order of the dynamics required by a pre-compensator to create the desired type. For post-compensation, Proposition 1 showed that, in general, a compensator of lower order could be used. In this section, a technique for pre-compensator design will be presented for the case where the restriction to diagonal compensation is lifted. The order of the compensator will be no greater than the order of the post-compensator of Proposition 1.

Mathematically, the problem of pre-compensator design to create a type $[\ell_1 \ \ell_2 \ \dots \ \ell_m]$ transfer function can be described as follows: Given a plant transfer function $G(s)$, find a dynamic compensator with transfer function $K(s)$ so that

$$G(s) K(s) = H(s) G'(s)$$

where

$$H(s) = \text{diag} \left[\frac{1}{\ell_1} \quad \frac{1}{\ell_2} \quad \dots \quad \frac{1}{\ell_m} \right]$$

and $G'(s)$ has no zeros at the origin.

The technique for solving this problem with reduced-order dynamics will be presented in the constructive proof to Theorem 2 which is the main result of this section. The idea of the theorem is that to increase any of the ℓ_1 by one in a type

$[\ell_1 \ \ell_2 \ \dots \ \ell_m]$ transfer function, a pre-compensator may be used which has only one finite pole, i.e., $K(s)$ can be found with order equal to one. The proof of the theorem requires the following two lemmas.

Lemma 1. Any column of an $m \times m$ polynomial matrix $D(s)$, which has elements which are not divisible by the linear term $(s - \alpha)$, can be brought to a form such that every element of the column except one is divisible by $(s - \alpha)$ via multiplication by a constant nonsingular upper triangular matrix M which operates on the left (i.e., a row operation matrix).

Proof. Without loss of generality, consider a single column of $D(s)$, any column q , which has elements $d_i(s)$ in row i . By assumption not all of the $d_i(s)$, $i = 1, 2, \dots, m$, are divisible by $(s - \alpha)$. Therefore there exists a greatest index k such that $d_k(s)$ is not divisible by $(s - \alpha)$. Polynomials are elements of a Euclidian ring [10] so $d_k(s)$ can be written as $d_k(s) = (s - \alpha) c_k(s) + r_k$ where $c_k(s)$ is a polynomial in s and r_k is a constant term. Also for every $j < k$, $d_j(s)$ can be written as $d_j(s) = (s - \alpha) c_j(s) + r_j$ where each $c_j(s)$ is a polynomial and each r_j is a constant. Now if a constant nonsingular upper triangular matrix M is formed which operates on the left of $D(s)$ so that $-(r_j/r_k)$ times row k is added to each row j for $j < k$, the resulting polynomials in row j of column q are

$$\begin{aligned} \tilde{d}_j(s) &= d_j(s) - (r_j/r_k) d_k(s) \\ &= (s - \alpha) c_j(s) + r_j - (r_j/r_k)(s - \alpha) c_k(s) - r_j \\ &= (s - \alpha) [c_j(s) - (r_j/r_k) c_k(s)] \end{aligned}$$

Thus $\tilde{d}_j(s)$ is divisible by $(s - \alpha)$. By assumption, $(s - \alpha)$ divides $d_i(s)$ for $i > k$ (M leaves row i , $i > k$, unchanged) and after operation on the left by M , $(s - \alpha)$ divides $\tilde{d}_j(s)$ for $j < k$. Thus only element $d_k(s)$ of column q is not divisible by $(s - \alpha)$. \square

Lemma 2. Let the $m \times m$ polynomial matrix $F(s)$ be the Smith form [8] of the $m \times m$ polynomial matrix $G(s)$ so that

$$F(s) = M(s) G(s) N(s)$$

where $M(s)$ is a unimodular $m \times m$ polynomial matrix representing row operations and $N(s)$ is a unimodular $m \times m$ polynomial matrix representing column operations, and $M(s)$ and $N(s)$ are not unique. Then for every unimodular row operation matrix $M'(s)$ of the type which adds a polynomial multiple of row i to row j with $i > j$, there exists a unimodular column operation polynomial matrix $N'(s)$ such that

$$M'(s) F(s) N'(s) = F(s)$$

Proof. By definition, $F(s)$ is a diagonal polynomial matrix with elements $f_{ii}(s)$ in the (i, i) position. The $f_{ii}(s)$ are such that $f_j(s)$ divides $f_k(s)$ whenever $j < k$. Thus if $M'(s)$ is a row operation polynomial matrix which adds $p(s)$ times row k to row j with $j < k$, the result is

$$M'(s)F(s) = \begin{bmatrix} \epsilon_1(s) & 0 & \cdot & \cdot & \cdot & 0 & 0 \\ 0 & & & & & & \\ \epsilon_j(s) & 0 & 0 & (p(s)\epsilon_k(s)) & 0 & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & & & \epsilon_k(s) & 0 & & \\ 0 & & & & \epsilon_m(s) & & \end{bmatrix}$$

Since $\epsilon_j(s)$ divides $\epsilon_k(s)$, the column operation matrix which adds $(-p(s)\epsilon_k(s)/\epsilon_j(s))$ times column j to column k is a polynomial matrix and gives the desired result. \square

It is remarked that by repeated application of this lemma, the result holds for any upper triangular row operation matrix $M''(s)$.

The main result of this section can now be proved.

Theorem 2. Given an $m \times m$ type $[\ell_1 \ell_2 \dots \ell_m]$ transfer function matrix $G(s)$, then for any i , $1 \leq i \leq m$, there exists an $m \times m$ transfer function matrix $K(s)$ of order equal to one such that the series $m \times m$ transfer function matrix $G(s)K(s)$ is type $[\ell_1 \ell_2 \dots \ell_i + 1 \dots \ell_m]$.

Proof. The proof is constructive. First, factor $G(s)$ into

$$G(s) = H(s) G'(s) = H(s) D^{-1}(s) N(s) \quad (4)$$

where

$$H(s) = \text{diag} \left[\frac{1}{\ell_1} \frac{1}{\ell_2} \dots \frac{1}{\ell_m} \right],$$

$G'(s)$ is type $[0 \ 0 \dots 0]$, and $D(s)$ and $N(s)$ are polynomial matrices obtained by a coprime factorization of $G'(s)$.

Now the problem may be restated as follows: Find an $m \times m$ transfer function $K(s)$ so that the series transfer function matrix $G'(s)K(s)$ is type $[0 \ 0 \dots 0 \ 1 \ 0 \dots 0]$ where the 1 is in an arbitrary specified position i . By Theorem 1 of [1], this is equivalent to finding $K(s)$ such that the coprime factorization

$$G'(s) K(s) = D'^{-1}(s) N'(s)$$

is such that every element of column i of $D'(s)$ is divisible by s .

The construction of $K(s)$ proceeds as follows. Let

$$\hat{N}(s) = P(s) N(s) Q(s) \quad (5)$$

where $\hat{N}(s)$ is the Smith form of $N(s)$; and $P(s)$ and $Q(s)$ are unimodular polynomial matrices. By the maximality of the ℓ_i , there exists at least one element in every column of $D(s)$ from Eq. (4) which is not divisible by s . With $P(s)$ defined by Eq. (5), find an upper triangular constant $m \times m$ matrix R so that every element of column i of the product $P(s) D(s)$ is divisible by s except one. This is always possible by Lemma 1 with $a = 0$. Let k indicate the row index of the element of column i which is not divisible by s , and let

$$\tilde{D}(s) = R P(s) D(s) \quad (6)$$

where every element of column i of $\tilde{D}(s)$ is divisible by s except the one in row k .

Now find a unimodular polynomial matrix $S(s)$ so that

$$R \hat{N}(s) S(s) = \tilde{N}(s) \quad (7)$$

This is always possible by Lemma 2 and the remark that followed it since $N(s)$ is in Smith form. In fact, $S(s)$ is determined uniquely by R .

Using Eq. (6), D^{-1} can be written

$$D^{-1}(s) = \tilde{D}^{-1}(s) [R P(s)] \quad (8)$$

Then with Eqs. (5) and (7), $N(s)$ can be written

$$N(s) = [R P(s)]^{-1} \tilde{N}(s) [Q(s) S(s)]^{-1} \quad (9)$$

Now construct $K(s)$ as

$$K(s) = [Q(s) S(s)] \tilde{K}(s) \quad (10)$$

where

$$\tilde{K}(s) = \text{diag} [1 \ 1 \dots 1 \frac{1}{s} \ 1 \dots 1]$$

with the $1/s$ factor in the k th position. Since both $Q(s)$ and $S(s)$ are polynomial matrices, $K(s)$ has order equal to one.

From Eqs. (8), (9) and (10), it follows that

$$\begin{aligned} G'(s)K(s) &= D^{-1}(s) N(s) [Q(s) S(s)] \tilde{K}(s) \\ &= \tilde{D}^{-1}(s) [R P(s)] [R P(s)]^{-1} \tilde{N}(s) [Q(s) S(s)]^{-1} [Q(s) S(s)] \tilde{K}(s) \\ &= \tilde{D}^{-1}(s) \hat{N}(s) \tilde{K}(s) \end{aligned} \quad (11)$$

Since both $\hat{N}(s)$ and $\tilde{K}(s)$ are $m \times m$ diagonal matrices whose elements are from a commutative ring, the matrices commute and Eq. (11) becomes

$$\begin{aligned} G'(s) K(s) &= \tilde{D}^{-1}(s) \tilde{K}(s) \hat{N}(s) \\ &= [\tilde{K}^{-1}(s) D(s)]^{-1} \hat{N}(s) \\ &= \tilde{D}^{-1}(s) \hat{N}(s) \end{aligned} \quad (12)$$

By construction of $\tilde{K}(s)$, $\tilde{K}^{-1}(s)$ is a polynomial matrix which multiplies row k of $\tilde{D}(s)$ by the factor s . Thus every element of column i of $\tilde{D}(s)$ is now divisible by s .

It remains to show that $\hat{D}(s)$ and $\hat{N}(s)$ in Eq. (12) are coprime. Then by Theorem 1 of [1], the present theorem is proved.

Now $\hat{D}(s)$ and $\hat{N}(s)$ are coprime if

$$\text{rank} [\hat{D}(s) : \hat{N}(s)] = m \quad (13)$$

for all s [8].

Consider first the case where $s = 0$. Since $\hat{N}(s)$ has no zeros at the origin, Eq. (13) is obviously true for $s = 0$.

Next consider the case where $s \neq 0$. Factor $[\hat{D}(s) : \hat{N}(s)]$ into the following:

$$\begin{aligned} [\hat{D}(s) : \hat{N}(s)] &= [\tilde{K}^{-1}(s) R P(s) D(s) : R P(s) N(s) Q(s) S(s)] \\ &= \tilde{K}^{-1}(s) [R P(s)] [D(s) : N(s)] \begin{bmatrix} I_m & 0 \\ 0 & Q(s) S(s) \end{bmatrix} K'(s) \end{aligned}$$

where

$$\begin{bmatrix} I_m & 0 \\ 0 & Q(s)S(s) \end{bmatrix}$$

is a $2m \times 2m$ matrix with the $m \times m$ block identity I_m and the $2m \times 2m$ matrix

$$K'(s) = \text{diag} [1 \ 1 \ \dots \ \frac{1}{s} \ 1 \ \dots \ 1]$$

has the $1/s$ factor in the $m+k$ position. For $s \neq 0$, $K^{-1}(s)$ and $K'(s)$ are nonsingular. Therefore,

$$[D(s); N(s)] = [RP(s)]^{-1} \tilde{K}(s) [\hat{D}(s); \hat{N}(s)] K'^{-1}(s)$$

$$\begin{bmatrix} I_m & 0 \\ 0 & Q(s)S(s) \end{bmatrix}^{-1}$$

and it follows that $\text{rank} [\hat{D}(s); \hat{N}(s)] \leq \text{rank} [D(s); N(s)] = m$ where the last equality is from the definition of the coprime factorization in Eq. (4).

Thus Eq. (13) is satisfied for all $s \neq 0$ and hence for all s . This completes the proof. \square

The proof of this theorem shows explicitly how to find $K(s)$. The construction may be complex since computation of the Smith form for large systems can be difficult. Whether or not $K(s)$ can be found as a proper rational matrix is an open question.

Example. Consider the problem of determining the pre-compensator which will render the type $[0 \ 0 \ 0]$ transfer function matrix

$$G(s) = \begin{bmatrix} 0 & \frac{1}{s+1} & \frac{s}{s+1} \\ 2 & 4 & 4 \\ \frac{6}{s+3} & 7 & \frac{8}{s+3} \end{bmatrix}$$

type $[0 \ 0 \ 1]$.

First $G(s)$ is factored into the form $G(s) = D^{-1}(s) N(s)$ yielding

$$G(s) = \begin{bmatrix} \frac{1}{s+1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} 0 & 1 & s \\ 2 & 4 & 4 \\ 6 & 7(s+3) & 8 \end{bmatrix}$$

Then $\hat{N}(s)$ can be found as $\hat{N}(s) = P(s) N(s) Q(s)$, viz.,

$$\hat{N}(s) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ -(7s+9) & -3 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & s \\ 2 & 4 & 4 \\ 6 & 7(s+3) & 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & -2(1-s) \\ 0 & 1 & -s \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -7s^2 - 9s - 4 \end{bmatrix}$$

Next, $P(s) D(s)$ is determined to be

$$P(s) D(s) = \begin{bmatrix} 0 & 1 & 0 \\ s+1 & 0 & 0 \\ -(7s+9)(s+1) & -3 & (s+1) \end{bmatrix}$$

For this example, it is immediately seen that R is the identity, and consequently, that $S(s)$ is also the identity. So $\tilde{K}(s) = \text{diag} [1 \ 1 \ 1/s]$. Then, $K(s)$ is found to be

$$K(s) = Q(s) \tilde{K}(s) = \begin{bmatrix} 1 & -2 & \frac{2(s-1)}{s} \\ 0 & 1 & -1 \\ 0 & 0 & \frac{1}{s} \end{bmatrix}$$

so that $G(s) K(s)$ is

$$G(s) K(s) = \begin{bmatrix} 0 & \frac{1}{s+1} & 0 \\ 2 & 0 & 0 \\ \frac{6}{s+3} & \frac{7s+9}{s+3} & \frac{-7s^2 - 9s - 4}{s(s+3)} \end{bmatrix}$$

which is of the desired type.

IV. CONCLUSION

As a sequel to [1], this paper has presented methods for specifying pre- and post-compensation to achieve a desired system type for linear multivariable servomechanisms. Sufficient conditions have been developed for both forms of compensation when the compensator is restricted to be characterized by a diagonal transfer function matrix. In the case of pre-compensation, a method of determining a reduced-order compensator has been derived and illustrated.

Further research is needed to extend the present results to permit the realization of compensators which simultaneously achieve a desired system type, guarantee closed-loop stability, and satisfy other performance specifications such as minimum integral-square-error. Some results in this direction are reported in [11].

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